Colonel Tennant has deposited the original negatives of the photographs taken on this occasion with the Astronomer Royal.

On Errors in Vlacq's (often called Briggs' or Neper's) Table of Ten-figure Logarithms of Numbers. By J. W. L. Glaisher, B.A., F.R.A.S., Fellow of Trinity College, Cambridge.

As, with the exception of Vega's *Thesaurus &c.*, the only complete table of ten-figure logarithms that has been published is the original one, partially calculated by Briggs and completed by Vlacq, the publication of errors detected in this work is of very considerable importance. Lists of errata have been given by Vlacq himself, by Vega, Sherwin, Lefort, and others, but it is not possible to find without comparison to what extent these lists are concurrent or supplementary; the main object of this note is, therefore, to give the results of an examination of these tables of errata, and to supplement them with some not previously published. In order to render clear what follows, it is necessary to premise some facts with regard to the original calculation of the logarithms of numbers.

The first table of logarithms to the base 10 was calculated by Henry Briggs, Savilian Professor of Geometry at Oxford, and published by him under the title, *Arithmetica logarithmica, sive logarithmorum chiliades triginta, pro numeris naturali serie crescentibus ab unitate ad* 20,000*: et a* 90,000 *ad* 100,000. . . . *Londini, excudebat Gulielmus Jones,* 1624.* This table contains the logarithms of the natural numbers from unity to 20,000, and from 90,000 to 100,000 to 14 places of decimals. There is thus left a gap from 20,000 to 90,000, which was filled up by Adrian Vlacq, who published at Gouda, in 1628, a table containing the logarithms of the numbers from unity to 100,000 to 10 places of decimals. Having calculated 70,000 logarithms and copied only 30,000, Vlacq would have been quite entitled to have called his a new work. He designates it, however, only a second edition of Briggs, the title running, *Arithmetica logarithmica, sive logarithmorum chiliades centum, pro numeris naturali serie crescentibus ab Unitate ad* 100,000. . . . . *Editio secunda aucta per Adrianum Vlacq, Goudanum.* . . . . *Goudæ excudebat Petrus Rammasenius,* 1628. This table of Vlacq's was published, with an English explanation prefixed, in London in 1631. The title of the English work is, *Logarithmicall Arithmetike;*

* Some copies, with the same title-page, contain an additional chiliad; viz., the logarithm of numbers from 100,000 to 101,000. The library of Trinity College, Cambridge, contains one of these copies, which belonged to Dr. Brinkley. In 1617 Briggs published his *Logarithmorum chilias prima.*

or, Tables of Logarithmes for absolute numbers, from an unite to 100,000. . . . . *London, printed for George Miller*, 1631.

Speaking of Briggs' *Arithmetica Logarithmica* of 1624, De Morgan, in his article on Tables in the English Cyclopædia, says, "After his [Briggs'] death, in 1631, a reprint was, it is said, made by one George Miller; the Latin title and explanatory parts were replaced by English ones—'Logarithmicall Arithmetike,' &c. We much doubt the reprint of the tables, and think that they were Briggs' own tables, with an English explanation prefixed in place of the Latin one. Wilson (in his *History of Navigation*, prefixed to the third edition of Robertson) says, that some copies of Vlacq of 1628 were purchased by our booksellers, and published at London with an English explanation premised, dated 1631. Mr. Babbage (to whose large and rare collection of tables we were much indebted in the original article) has one of these copies; and the English explanation and title is the same as that which was in the same year attached to the asserted reprint of Briggs. We have no doubt that Briggs and Vlacq were served exactly in the same manner." On referring to Robertson (Fourth Edition, p. xvi.), I find no further information than that contained in the above extract. That the above suggestion of De Morgan's is correct, and that Miller's and Vlacq's tables are both printed from the same types, is evident on examination. Miller's copies are not quite so rare as they might be inferred to be from the above remarks; there is one in the University Library, Cambridge, and one in the library of Trinity College, Cambridge; while the library of the Royal Observatory at Greenwich contains two; there is, however, no copy in the libraries of the Royal Society or the Royal Astronomical Society. On the fly-leaf of the first of these copies (viz. that in the University Library) there is written in pencil, "N.B. These tables were edited by Vlacq and printed at Gouda." Three out of the four copies mentioned (the Trinity one is the exception) contain, after the English introduction and before the tables, a title-page, on which is printed, *Tafel der Logarithmi voor de Ghetallen van* 1 *af tot* 100,000, while the corresponding page in Vlacq's table is inscribed, *Chiliades centum Logarithmorum pro numerus* (sic) *ab unitate ad* 100,000. This shows the Dutch origin of Miller's table; and it is besides curious, as it seems to imply that Vlacq meditated a Dutch translation of his work, but that the tables intended for the purpose were bought and published by George Miller. A careful examination of any one page of Vlacq's or Miller's copies is sufficient to show that they were printed from the same types; a good confirmation is afforded by the following error: in the numbers 3376, 3377 . . . . 3380 were printed instead of 4776, 4777 . . . . 4780; to remedy this a small piece of paper, on which the latter numbers were printed, was stuck over the former, and this occurs in both Vlacq's and Miller's copies. This and similar facts place beyond all doubt the identity of the tables, whether published in Gouda or London, and to

establish this fact is important, as we are thus assured that the same errata-list suffices for both. It is this table (which in Miller's copies, from the occurrence of both names on the title-page, is generally called after Briggs or Neper, and in the original copies after Vlacq) which was referred to at the beginning of this note.

In 1631 Vlacq published his *Trigonometria Artificialis*. This work contains, among other tables, the logarithms of the numbers from unity to 20,000, printed also (with the exception of the last sheet referred to further on) from the same types.

No further calculation of logarithms of numbers took place till the end of the last century, when the great French manuscript tables (the *Tables du Cadastre*) were computed under the direction of Prony. These, as is well known, have never been published.

The lists of errata which will be particularly referred to are,—that prefixed to Vlacq's own table in the *Arithmetica Logarithmica*,* that given by Vlacq on the last page of the table of the logarithms of numbers in the *Trigonometria Artificialis*, that given by Sherwin, and the elaborate table of M. Lefort, in t. IV. of the *Annales de l'Observatoire de Paris*, pp. [148]-[150], which was obtained by comparison with the *Tables du Cadastre*. This list also contains the errors given by Vega in the *Thesaurus logarithmorum completus*. It is intended to be supplementary to Vlacq's list in the *Arithmetica*, so that the two taken conjointly should give all the errors. One of the copies in the Greenwich Observatory library belonged to Maskelyne, and the following note is written on the cover: "All the errata have been corrected that had been discovered by, and are contained in William Gardiner's book (the author of the logarithm sines, &c., to every 10 seconds). Others, 22 in number, marked T, have been corrected, which were discovered by Michael Taylor, author of the logarithm sines and tangents to every second of the quadrant. Nevil Maskelyne, Astron. Regius, March 23, 1790."

I do not know where the errata found by Gardiner and Taylor were published, if at all: Gardiner's are not given in his tables. It is very probable that Taylor's were communicated personally to Maskelyne; it is worth notice that though the above note is dated 1790, yet Taylor's logarithm tables were not published till 1792. I have made no special search for either of the lists referred to, as the correction of the errors in the volume was sufficient for the purpose I had in view; viz., the discovery of errors not included in the lists of Vlacq and Lefort. The result of a careful comparison between Maskelyne's copy and Vlacq's and Lefort's list of errata was the detection of the following seventeen errors, not given by Vlacq in the *Arithmetica*, nor by Lefort.

* This list does not occur in Miller's copies.

Number.	Error.	Correction.
1360	89083	89084
1622 Diff.	26,76609	26,76699
2154	56970	56990
3192	13,62358	13,60358
7117 Diff.	6,10169	6,10179
9329 Diff.	4,75506	4,65506
9610 Diff.	4,51869	4,51896
*11275	65506	65505
11293	93280	93281
11699	87409	87410
18723	54375	54373
18724	86323	86325
23999	4,34019	4,38019
24580	18785	18786
53250 Diff.	81555	81557
*54040	53404	53403
64818	56295	56265

In the case of the two numbers marked with an asterisk the error is of no importance whatever, and not worth correcting; thus, the logarithm of 11275 is 4·05211, 65505, 49998, 14 . . ., and it is a matter of indifference whether the tenth figure of the mantissa be increased or not; log. 54040 = 4·73271, 53403, 49992, 98 . . ., and the same remark applies. These two logarithms were easily calculated by means of the logarithms up to 1200 to 20 places given by Callet, since $11275 = 25 \times 451$ and $54040 = 280 \times 193$.

On subtracting the logarithm of 14485 from that of 14486, the difference is 299814 instead of 299813, and Maskelyne's copy is accordingly so corrected; the book is, however, correct, as on referring to Briggs' table of 1624 the true difference is 2998132 . . ., the unit difference being due to the correction of the last figures of the logarithms in Vlacq. All the errata in the above list I have examined carefully, and satisfied myself that they are accurate. The errors corresponding to the numbers 1622 and 7117 also occur in Briggs' table of 1624.

The following are errata in Lefort's errata-table. Corresponding to the number 63747 the number in the error column should be 974 instead of 947. For the number 53053, the correction there given is 3 instead of 2 in the last figure, but the logarithm ends with a 1 and not a 3, and on calculating it from Callet ($53053 = 77 \times 689$) I find that it is correct in the book. The correction probably has reference to some other number. For the number 50996 the correction 4 is printed so faintly as to be almost illegible, and for 16399 and 16699 the letter N is omitted.

Vlacq's errata-list in the *Arithmetica* contains no error; the

correction belonging to the number 14694 is superfluous in all the copies I have seen, the difference being printed correctly.

The following facts may be of interest: Vlacq's list in the *Arithmetica* contains 120 errata (106 of which correspond to numbers below 20,000). These are all corrected in Maskelyne's copy.* Lefort's list contains 452 errata, 301 of which are marked with an asterisk, to imply that they were given by Vega. Out of the 452, however, 287 may be called unimportant, denoting by this word errors which only alter by unity the last figure of the logarithm. There are 265 errors given by Lefort, not corrected in Maskelyne's copy, of these 222 were unimportant, and only 43 serious. There were, however, three corrections in Maskelyne's copy that were inaccurate, the original number being correct. The list in the *Trigonometria* (which only gives the logarithms of numbers from unity to 20,000) contains 79 errata in the logarithms and 40 in the differences; of the former three are not given in the *Arithmetica* (viz., those for the numbers 6197, 9182, 9429); they are to be found in Lefort. Of the errors in the differences 15 are new, while 5 (viz., those for the numbers 11400, 11499, 12582, 13639, 19081) which are given in the *Arithmetica* are omitted. Why these were not included I cannot guess; they are all important and uncorrected in the *Trigonometria*. The sheet (signature M) giving the numbers beyond 19500 is reprinted for the *Trigonometria*, and the error in 19960 (number) is corrected.

At the end of the preface to Sherwin's Tables† is a list of errata, those not given by Vlacq being marked with an "a." The error for the number 2167 should not be so marked, as it is to be found in both the *Arithmetica* and *Trigonometria;* three (viz., those for the numbers 6197, 9182, 9429) are in the *Trigonometria* and not in the *Arithmetica;* they therefore appear also in Lefort. The corrections belonging to the numbers 56359 and 57756, though marked with an "a," are not to be found in Lefort; they are, however, superfluous, as the logarithms are printed correctly in the copies I have seen: it is probable that they referred to indistinct figures in the copy Sherwin made use of. On the whole, Sherwin added twenty-one errata, of which one was not new and two were superfluous; the remaining eighteen are to be found in Lefort. Sherwin's list only gives errata in logarithms, not in differences. I have not made an examination of Vega's errata-list, feeling no doubt that this was completely done by Lefort.

Having found an error in one of the numbers, which was not

* In one instance the correction was wrongly made.

† Of Sherwin's Tables, De Morgan remarks in the article previously cited. "Second edition, 1717; third revised by Gardiner, and the best, 1742; fifth and last, 1771, very erroneous—the most inaccurate table Hatton ever met with." My father's copy, the errata-list in which I used in the above comparison, is dated 1726, and contains a list of "errata for the second edition of Sherwin's Mathematical Tables," by Gardiner.

given in any of the above errata-lists, it occurred to me that errors in the numbers would have stood a far better chance of escaping detection than errors in the logarithms, as the latter would be examined more or less by all succeeding editors of logarithmic tables; I have therefore made an examination of the numbers from 1 to 100,000 to discover misprints. The numbers from unity to 21,500 I examined myself, the rest were done for me by an assistant. The result is the discovery of errors in the following numbers:—

969	*16699
2081	19960
2082	34163
2260	49831
2278	*66150
3500	*66789
7368	*66790
12301	*69489
12345	*69490
16327	77052
*16399	

It is unnecessary to give the errors in the above numbers; the correction being, of course, evident at sight. The corrections for the seven numbers to which the asterisk is prefixed are contained in Lefort. Of the above twenty-one errors, twelve are corrected in Maskelyne's copy. It is probable that Vlacq's list in the *Arithmetica*, Lefort's, and the two lists in this paper, taken conjointly, contain nearly, if not quite, all the errors in Vlacq's tables. The total number is thus 603, of which about one-half are unimportant; the tables contain over 2,100,000 figures liable to error, so that there is one error, on the average, to every 3500 figures, or an important error to every 7000. Considering that the tables were the result of an original calculation, and that many of the errors were caused by the slipping of figures after the proofs had left the author's hands, the great accuracy of the table must be admitted by all. Two trivial errors may be noted; viz., Chilias 28 should be Chilias 38 in the first column of the page beginning with the number 37051, and the number in the corner of the page beginning with the number 66001 should be 66151 instead of 66251. In the course of the examination of the numbers, 49 were marked as containing figures so imperfectly printed, that they might lead to error; on referring to another copy, this was the case with but 25 of these; viz., 576, 4576, 7106, 16826, 19650, 21286, 24077, 30420, 31176, 31226, 33326, 37088, 41426, 45876, 61226, 61526, 66876, 66896, 81026, 83864, 96903, 97318, 97326, 97328, and 98280. Of course, in many copies these may be printed quite legibly, but the occurrence of imperfections in them in two copies renders it probable that they

may be similarly imperfect in others. It might also be advisable in any copy to ascertain that the slip of paper, previously noticed, containing the numbers 4776–4780, had not come off and been lost; but in point of fact the only part of the table ever used is that for the numbers 10,000 to 100,000, for which the characteristic is 4. It may be remarked that several of the alterations in Maskelyne's copy are made in the text, and these at first sight might give the impression that all the copies were not printed from the same types; such, however, is not the case.

The increase of the last figure in tables, when the succeeding figures are greater than 500 . . ., seems to deserve more attention than it has received. Errata, such as some noticed in this communication, where the succeeding figures are 499 . . ., are by no means uncommon; and it appears that the discoverers of them imagine they are doing some service by noting them. Take, for example, one of the cases in this note: the figures starting from the tenth are 5 49998 . . .; if we take 5 as the tenth figure, the error is 49998 . . ., if 6, the error is 50002, differing by 00004. Now, as our table only professes to give 10 places correctly (regard being paid to the magnitude of the figure in the eleventh place), a difference in the fifteenth place does not come in question at all: 5 and 6 are both equally correct; they only differ by quantities, which throughout all the rest of the table we agree to neglect. It is a matter of regret that all such valueless refinements are not avoided by the author always explaining the exact convention on which the last figure is increased. A very convenient arrangement would be to understand that when x figures of a number were tabulated, the error was less than 6 in the next figure; or, if the calculator wished to be more accurate, 5 6 in the next two figures. To obtain a table of x figures, it is usual to calculate $x + 1$, or $x + 2$ figures, and the inconvenience of extending the calculation further in the particular case when the next figure is 5, or the next two 50, is, in many cases, excessive, and as the result is of no additional value when obtained, a figure "wrong" under these circumstances ought not to be styled an error. Probably a good many of Lefort's errata are of this class. Babbage, in the introduction to his well-known table of seven-figure logarithms, states, that in ninety-three instances the next three figures in Vega were 500, and that in all these cases the logarithms were carried to more than ten places to determine whether the figures were really 500 . . . or 499 . . ., and decide whether the least figure was to be increased or not. This appears to me to have been quite needless. It sets up an unnecessary and artificial standard of accuracy for the numbers whose seventh, eighth, and ninth figures happen to be 4,9,9 or 5,0,0. To the user of a table of seven-figure logarithms it is a matter of really no importance whether his error is 499 or 501; he is content to make an error of 5, and an additional error of $\pm$ 001 is of no consequence.

It is not a little remarkable that the most accessible table of

ten-figure logarithms we possess should have been published nearly 250 years ago. This places their use out of the power of all who have not access to an important library. In mathematical computations, ten-figure logarithms are very often wanted, even when only 5 or 6, or even a less number of figures of the ultimate function are intended to be tabulated. Most functions are expansible in an ascending (convergent) series of the form $A_0 + A_1 x + A_2 x^2 + \ldots$, and a descending (semi-convergent) series of the form $\frac{B_1}{x} - \frac{B_2}{x^2} + \frac{B_3}{x^3} - \ldots$. The former is usually very convenient for values of x less than unity, and moderately so till $x = 3$ or 4 perhaps, while the latter may, from the convergent portion, give the required number of places from, say, $x = 12$ or 20; but between the two practicable ranges of the two series there is usually a gap, and it is in cases such as these for the calculation of the intermediate values (if there is no continued fraction available for the purpose) that ten-figure logarithms are of the greatest use.

The title of Vega's work is *Thesaurus Logarithmorum completus ex Arithmeticâ logarithmicâ et Trigonometriâ artificiali Adrian Vlacq . . . a Georgio Vega. Leipsiæ*, 1794 (folio). The whole work contains 684 pp. of tables, of which the logarithms of numbers from unity to 100,100 occupy 305 pp. The arrangement is similar to that generally in use for seven-figure tables, and a great saving of room is effected: this advantage, however, is neutralised in the present work for the computer who only requires the logarithms of numbers by the other tables that are added. It is to be inferred from the preface that the publication of the tables was due to the great scarcity and expense of Vlacq's works; but, at all events in England, Vega's work is more scarce now than the latter. Many abridgments of Vega have been published.

Mention must be made of a work published last year; viz. *Tables de Logarithmes vulgaires à dix decimales, construites d'après un nouveau mode par S. Pineto . . . S. Petersbourg*, 1871, which by a new and ingenious arrangement in effect gives a complete table of ten-figure logarithms in 56 octavo pages. It does not seem quite so convenient as the older tables, and I imagine any one who can obtain the latter will prefer to use them.

Mr. Sang, of Edinburgh, has announced the intended publication of a nine-figure table of logarithms of numbers from unity to a million, which will be of great value to mathematical calculators; but, even with this work, the additional figure in Vlacq will prevent his table from being superseded entirely.

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November, 1789), "I should certainly have been able to announce its existence as early as August 19, 1787, when at $22^h\ 18^m\ 56^s$, I saw and marked it down as probably a sixth satellite, then about 12° past its greatest preceding elongation. . . . In the year 1788 very little could be done towards a discovery as my 20-feet speculum was so tarnished by zenith sweeps that I could hardly see the *Georgian* satellites. In hopes of great success with my 40-feet speculum I deferred the attack on *Saturn* till that should be finished, and having taken an early opportunity of directing it to *Saturn*, the very first moment I saw the planet, which was the 28th of last August, I was presented with a view of six of its satellites in such situations, and so bright as rendered it impossible to mistake or not to see them; and also on the 17th Sept. I detected the seventh satellite when at its greatest preceding elongation." In ascertaining the period of the sixth satellite, Herschel states that he used the 19th August, 1787, as a starting-point. Later he states that the seventh satellite "appears in the 40-feet, no bigger than a very small lucid point," yet he says, "I see it very well with the 20-feet reflector, to which the exquisite figure of the speculum not a little contributes." This is the account referred to in the former part of this paper.

It seems demonstrated then that though the satellite was seen in 1787 (as Hind also mentions), with the 20-feet telescope, it was not discovered in any proper sense of the word until August 28th, 1789, the instrument of the discovery being the 40-feet telescope. If the observation of 1787 is to be regarded as the discovery of the satellite, then by parity of reasoning Herschel did not discover *Uranus* nor did Galle observationally discover *Neptune*.

Note on the Densities of Jupiter's Satellites.

By Richard A. Proctor, B.A.

Incorrect values of the densities of *Jupiter's* satellites have somehow found their way into our text-books of astronomy, and have been repeated from one to another. They have led to erroneous assumptions respecting the condition of these bodies.

Thus, in Lardner's *Handbook*, we find the following table: —

Satellite.	Mass, that of Jupiter = 1.	Mass, that of Earth = 1.	Density, that of Earth = 1.	Density, that of Water = 1.
I	0·0000173	0·00520	0·02016	0·1143
II	0·0000232	0·00698	0·03015	0·1710
III	0·0000885	0·02663	0·06984	0·3960
IV	0·0000427	0·01285	0·03925	0·2225

It is strange that, though accepting different values of the satellites' diameters, Mr. Chambers, in his *Descriptive Astronomy*, gives the same values for the densities, only omitting the last decimal figure.